MCS-031

MCA (Revised)

Term-End Examination

June, 2008

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest. Credit will be given for to-the-point answers.*

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1. (a) (i) For a given problem P, two algorithms A1 and A2 have time complexities T1(n) and T2(n) respectively in terms of size n where

$T1(n) = 4n^5 + 3n$ and

$T2(n) = 2500n^3 + 4n$.

Find the range of n for which A1 is more efficient than A2. 4

(ii) Find the asymptotic upper bound for the recurrence equation

$T(n) = T(n/2) + T(n/4) + T(n/8) + n$

with boundary condition $T(1) = 1$. 4

(b) (i) Write an algorithm for insertion sort on an array of size n. 4

(ii) Estimate the best case running time and worst case running time for this insertion sort algorithm. 4

(c) (i) Define fractional knapsack problem. Give a greedy algorithm for this problem. 4

(ii) List the difference between greedy technique and dynamic programming technique. 4

(d) Explain how using dynamic programming reduces the complexity of a simple algorithm. Also explain the matrix chain multiplication algorithm in this context. 4+4

(e) (i) What are intractable problems ? 4

(ii) Explain the steps in establishing the NP-completeness of a problem. 4

2. (a) (i) Give a divide and conquer based algorithm to find the i^{th} smallest element in an array of size n. 5

(ii) Derive the running time of your algorithm. 5

(b) The binomial coefficient may be defined by the following recurrence equation $C(n, 0) = 1$ and $C(n, n) = 1$ for $n \geq 0$

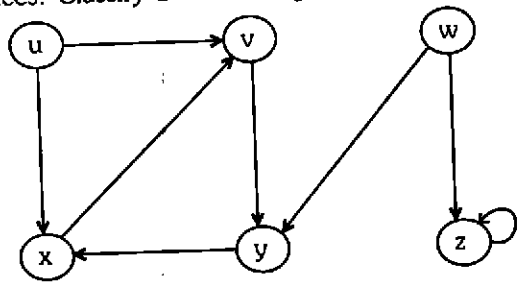
$C(n, k) = C(n - 1, k) + C(n - 1, k - 1)$ for $n > k > 0$.

(i) Draw the recursion tree for calculating $C(6, 4)$. 2

(ii) Write a recursive function to generate $C(n, k)$. 3

(iii) Give an algorithm based on dynamic programming technique to solve $C(n, k)$. 5

3. (a) Obtain the DFS Tree for the following graph. Show the discovery time and finishing time for each of the vertices. Classify the back edges and cross edges. *10*



(b) Use PRIMS algorithm to find Minimum Spanning Tree for the following weighted adjacency matrix representation of an undirected graph G. *10*

	a	b	c	d	e	f	g
a	–	4	–	–	–	8	–
b	4	–	8	–	–	11	–
c	–	8	–	4	–	–	2
d	–	–	4	–	2	–	–
e	–	–	–	2	–	1	6
f	8	11	–	–	1	–	7
g	–	–	2	–	6	7	–

4. (a) (i) What do you mean by overlapping subproblems ? Explain w.r.t. binomial coefficient equation in Question 2(b). *5*

(ii) What are Regular expressions ? Write a regular expression over $\Sigma = \{0, 1\}$ to generate all strings that end with two ones. *5*

(b) Explain why Quick sort algorithm is considered the best. Also explain how worst case time complexity can be reduced by using randomization quick sort. 5+5

5. (a) (i) Define NP-complete problems. 5

(ii) Write the algorithm for greedy approach for Huffman code. 5

(b) (i) Define a Turing Machine. 5

(ii) Construct a Turing Machine to accept all languages of palindromes on alphabet $\Sigma = \{a, b\}$. 5